Eur. Phys. J. A **31**, 424–428 (2007)
DOI 10.1140/epja/i2006-10189-2

THE EUROPEAN PHYSICAL JOURNAL A

Special Article – QNP 2006

The neutrinoless double-beta decay: A test for new physics

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Received: 8 October 2006
Published online: 22 February 2007 –

Abstract. The neutrinoless double-beta decay is not allowed in the Standard Model (SM) but it is allowed in most Grand Unified Theories (GUTs). The neutrino must be a Majorana particle (identical with its antiparticle) and must have a mass to allow the neutrinoless double-beta decay. Apart of one claim that the neutrinoless double-beta decay in ^{76}Ge is measured, one has only upper limits for this transition probability. But even the upper limits allow to give upper limits for the electron Majorana neutrino mass and upper limits for parameters of GUTs and the minimal R-parity violating supersymmetric model. One further can give lower limits for the vector boson mediating mainly the right-handed weak interaction and the heavy mainly right-handed Majorana neutrino in left-right symmetric GUTs. For that, one has to assume that the specific mechanism is the leading one for the neutrinoless double-beta decay and one has to be able to calculate reliably the corresponding nuclear matrix elements. In the present contribution, one discusses the accuracy of the present status of calculating the nuclear matrix elements and the corresponding limits of GUTs and supersymmetric parameters.

PACS. 23.40.-s β decay; double β decay; electron and muon capture – 21.60.-n Nuclear structure models and methods – 12.60.-i Models beyond the standard model

1 Physics beyond the standard model and the double-beta decay

In the standard model the neutrinoless double-beta decay is forbidden but it is allowed in most Grand Unified Theories (GUTs) where the neutrino is a Majorana particle (identical with its antiparticle) and where the neutrinos have a mass.

In GUTs, each beta decay vertex in fig. 1 can occur in eight different ways (see fig. 2).

The hadronic current changing a neutron into a proton can be left or right handed, the vector boson exchanged can be the light one W_1 or the heavy orthogonal combination W_2 mediating mainly a right-handed weak interaction and two different leptonic currents changing a neutrino into an electron. So the simple vertex for the beta decay can occur in eight different ways.

With two such vertices and the exchange of a light or a heavy mainly right-handed Majorana neutrino one already has 128 different matrix elements describing the neutrinoless double-beta decay [1] (see fig. 3).

At the R-parity violating vertex (see fig. 4) of the up- and the down-quarks and the SUSY electron $\tilde{e}$, one has the R-parity violating coupling constant λ'_{111} which is new compared to the standard model [2]. The formation of a pion by a down- and an up-quark produces a long-range

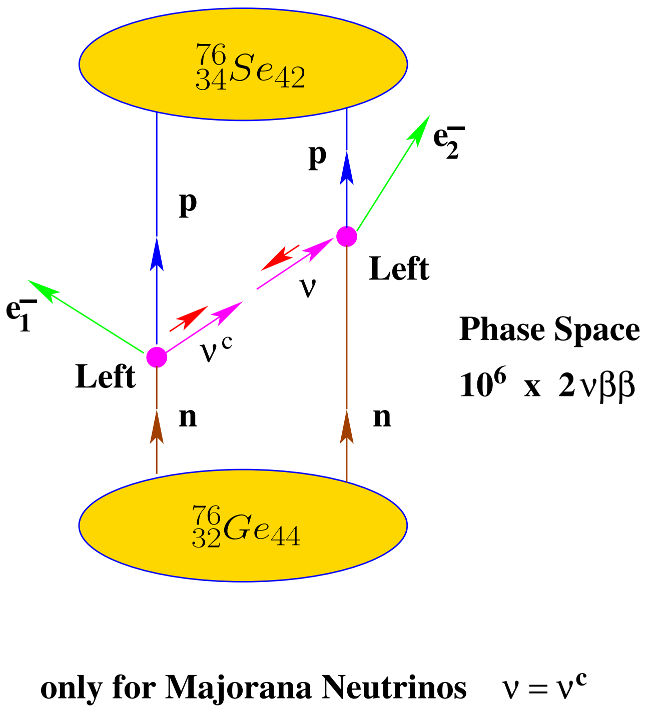

Fig. 1. Neutrinoless double-beta decay of ^{76}Ge through ^{76}As to the final nucleus ^{76}Se. The neutrino must be a Majorana particle that means identical with its antiparticle and must have a mass to allow this decay.

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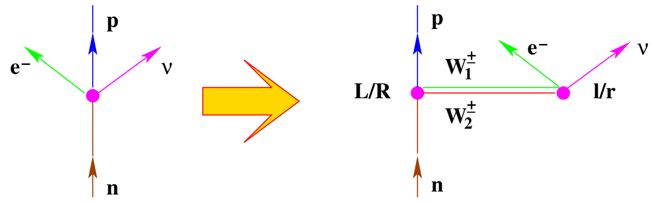

$$l/r = \bar{e}\,\gamma^{\mu}\left(1 \mp \gamma^{5}\right)\nu$$
$$L/R = p^{+}\,(g_V \mp g_A\,\vec{\sigma})\,n$$
$$g_V = 1 \;\; ; \;\; g_A = 1.25$$

Fig. 2. The single-beta decay changes a neutron into a proton by the emission of an electron and (in the standard model) an antineutrino. The microscopic version is shown on the right side of the figure.

$$\hat{H}^{\rm GUT}_{\rm Weak} = \frac{G_F \cos\vartheta_c}{\sqrt{2}}\left[1\cdot l\cdot L + t\,g\,\vartheta\; r\cdot L + t\,g\,\vartheta\; R\cdot l + \frac{M_1^2}{M_2^2}\,r\cdot R\right]$$

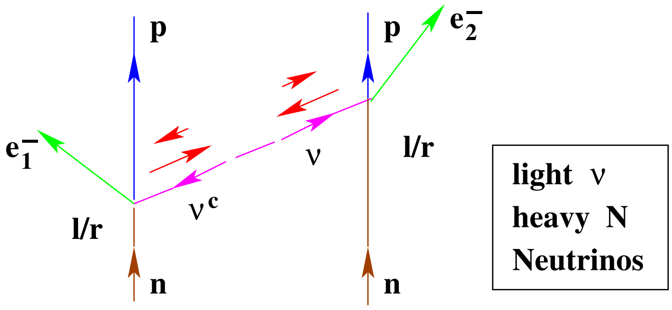

Fig. 3. Diagram of the double-beta decay.

neutrinoless double-beta decay transition operator. Due to the short-range Brueckner repulsive correlations between two nucleons, this is increasing the transition probability by a factor 10000. The upper limit derived from upper limits of the neutrinoless transition probability for λ'_{111} is therefore more stringent and one can derive from the neutrinoless transition probability in ^{76}Ge an upper limit for $|\lambda'_{111}| < 10^{-4}$.

To calculate the neutrinoless double-beta decay transition probability, we use Fermi's Golden Rule in second order:

$$\begin{aligned}
T &= \sum_k \frac{\langle f|\hat{H}_W|k\rangle\langle k|\hat{H}_W|i\rangle}{E_i - E_k},\\
E_i &= E_{0+}(^{76}\mathrm{Ge}),\\
E_k &= E_k(e^-) + E_k(\nu) + E_{\rm nucleus}(^{76}\mathrm{As}; k),\\
T &= M_m\langle m_\nu\rangle + M_\theta\langle \mathrm{tg}\,\vartheta\rangle\\
&\quad + M_{WR}\left\langle\left(\frac{M_2}{M_R}\right)^2\right\rangle\\
&\quad + M_{SUSY}\lambda'^2_{111} + M_{VR}\left\langle\frac{m_p}{M_{VR}}\right\rangle,\\
w &= \frac{2\pi}{\hbar}|T|^2 p_f \le 4.4\cdot 10^{-33}\ [\mathrm{s}^{-1}]. \qquad (1)
\end{aligned}$$

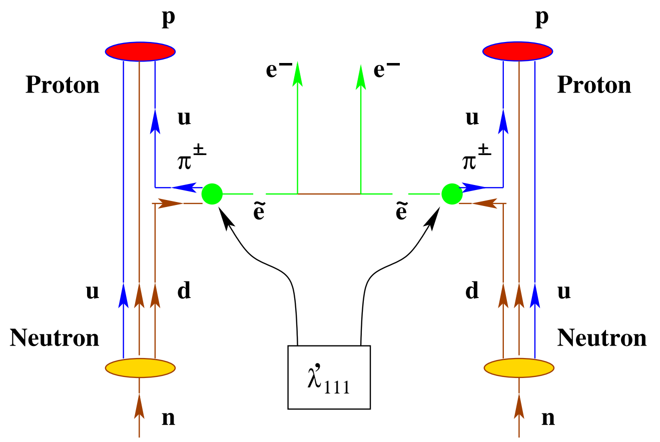

Fig. 4. Matrix element for the neutrinoless double-beta decay within R-parity violating supersymmetry.

Here $|k\rangle$ are the intermediate nuclear states in ^{76}As with a Majorana neutrino and one electron. The sum over k includes also an integration over neutrino energies. E_{0+} (^{76}Ge) is the ground-state energy of ^{76}Ge.

To calculate the nuclear matrix elements, the most reliable method has turned out to be the Quasiparticle Random Phase Approximation (QRPA) to calculate in the example of ^{76}Ge the wave function of the initial ^{76}Ge, the excited states of the intermediate nucleus ^{76}As and the ground state of the final nucleus ^{76}Se [3].

The QRPA approach describes the intermediate excited nuclear states $|m\rangle$ for example in ^{76}As as a coherent superposition of two quasiparticle excitations and two quasiparticle annihilations relative to the initial ground state (in our example) of ^{76}Ge:

$$\begin{aligned}
a_i^+ &= u_i c_i^+ - v_i c_{\bar{i}},\\
A_\alpha^+ &= [a_i^+ a_k^+]_{J\alpha},\\
Q_m^+ &= \sum_\alpha \left[X_\alpha^m A_\alpha^+ - Y_\alpha^m A_\alpha\right],\\
|m\rangle &= Q_m^+|g\rangle. \qquad (2)
\end{aligned}$$

The QRPA equation for the determination of the intermediate states by X_α^m and Y_α^m is derived from the many-body Schroedinger equation by using quasiboson commutation relations for the quasiparticle pairs $A_\alpha^\dagger$. One uses therefore for deriving the QRPA equations that two quasiparticle states behave like bosons. This is a usual approximation one often is using in physics. For example, one describes a pair of quarks and antiquarks as a meson and treats it as a boson, although it consists out of a fermion pair.

To test the quality of this approximation for the two neutrino and the neutrinoless double-beta decay, we include the exact commutation relations of the fermion pairs at least as ground-state expectation values. This is called renormalized-QRPA (R-QRPA) applied in [4] to the two neutrino double-beta decay and in [5] for the first time to the interesting neutrinoless double-beta decay.

The R-QRPA includes the Pauli principle into the QRPA and reduces by that the number of quasiparticles

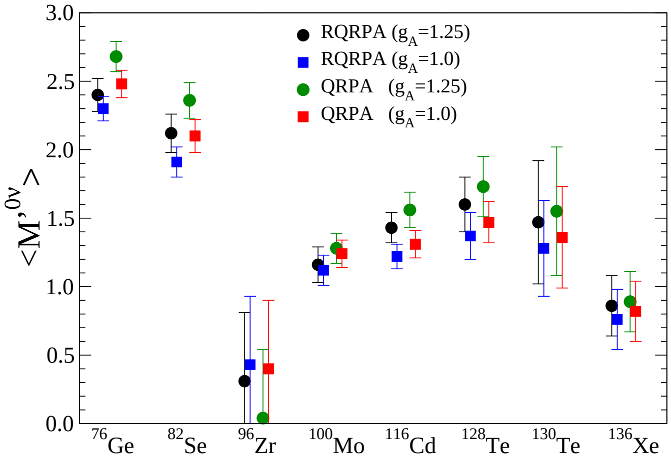

Fig. 5. Neutrinoless double-beta decay matrix elements at the Majorana neutrino mass calculated for different initial double-beta decay nuclei (for more details see text).

in the ground state. R-QRPA stabilizes the intermediate nuclear wave functions compared to the QRPA approach. One is relatively close to a phase transition to stable proton-neutron Gamow-Teller correlations. Since they are not included in the basis, the QRPA solution is collapsing if the 1^+ proton-neutron nucleon-nucleon two-body matrix elements are increased. One usually studies this by multiplying all nucleon-nucleon particle-particle matrix elements of the Brueckner reaction matrix of the Bonn (or the Nijmegen or Argonne) potential with a factor g_{pp} (typical: $g_{pp} \approx 0.9$). With an increasing g_{pp} the QRPA solution collapses like a spherical solution is collapsing if a nucleus gets deformed by increasing the quadrupole force. The inclusion of the Pauli principle in the R-QRPA approach moves this instability to much larger nucleon-nucleon matrix elements (larger factors g_{pp}) and the agreement for the two neutrino double-beta decay with the experimental value is more in the range of $g_{pp} = 1$.

Figure 5 shows the neutrinoless double-beta decay matrix elements calculated in 36 different ways for each initial nucleus indicated in the figure [6]. The approach includes three different forces (Bonn, Nijmegen, Argonne) and three different basis sets (about two oscillator shells, about three oscillator shells and about five to six oscillator shells) and four different approaches QRPA and the renormalized QRPA (RQRPA) with two different axial vector coupling constants $g_A = 1.25$ and with the quenched value $g_A = 1.0$. In each of the 36 calculations for each nucleus, the factor g_{pp} in front of the two nucleon-nucleon matrix element is adjusted to reproduce the experimental value of the two neutrino double-beta decay. The error bars given in the figure include the 1σ error of the theory plus the experimental error of the two neutrino double-beta decay transition probability. In several nuclei, like ^{76}Ge and ^{100}Mo the errors are as small as $\pm 15\%$. In most cases the error is less than $\pm 30\%$. But for ^{96}Zr one has an error of the neutrinoless double-beta decay matrix element of $\pm 100\%$. The main part of the error indicated in fig. 5 are the experimental error of the two neutrino double-beta decay probability. So when this measurements are improved, one is also automatically improving the prediction of the neutrinoless double-beta decay matrix elements. Results beyond the Standard Model using the most restrictive limit for the neutrinoless double-beta decay (hep-ph/0512263) with our matrix elements are given in table 1.

Table 1. This table gives limits for the lifetime and some parameters derived from data of ^{76}Ge. A lower limit of the experimental lifetimes for the 0ν double-beta decay is indicated in the second row with the experimental reference in the third row. The fourth row indicates the upper limit of the Majorana neutrino mass derived with the help of our QRPA matrix element in an intermediate basis (three oscillator shells). The fifth row gives the ratio of the proton mass over heavy Majorana neutrino mass for ^{76}Ge. This ratio is smaller than 0.79 which means that the averaged heavy Majorana neutrino, which is mainly right-handed, should have a mass larger than 1.2 GeV. The last row gives the upper limit of the R-parity violating coupling constant λ'_{111}. The upper limit of about 10^{-4} derived from ^{76}Ge is by one order of magnitude more stringent than other published values.

Nucleus	^{76}Ge
$\tau^{1/2}$ (exp) [y.]	$> 1.9\ 10^{25}$
Klapdor *et al.*	hep-ph/0512263
$\langle m \rangle$ [eV]	< 0.47
$m(p)/M(\nu)$	< 0.79
$\lambda'_{111}[10^{-4}]$	< 1.1

The present calculation includes the higher-order currents according to Towner and Hardy [7] and short-range correlations in the neutrinoless double-beta decay transition probability with a Jastrow factor.

The results of other groups seem partially not to agree with our results given above:

Muto's [8] results agree with our results if we include into his calculations the higher-order currents. The induced pseudoscalar current required by PCAC reduces his results by 30%. In addition we have a nuclear radius $R = 1.1 \cdot A^{1/3}$ [fm] of the Woods-Saxon potential compared to $R = 1.2 \cdot A^{1/3}$ [fm] of Muto. This difference reduces his values further by 10%. If we reduce his values for ^{76}Ge by 40% his values $M^{0\nu} = 4.59$ (QRPA) and 3.88 (R-QRPA) are reduced to 2.76 (QRPA) and 2.34 (R-QRPA), which agree nicely with our result of 2.68 (QRPA) and 2.41 (R-QRPA).

Civitarese and Suhonen have consistently higher values.

The calculation of Civitarese and Suhonen [9] are different from ours in several respects (see table 2):

1. They have fitted the factor g_{pp} not to the two-neutrino double-beta decay but to three known single-beta decay transition probabilities from a 1^+ ground state in intermediate nuclei to the final nuclear ground state.
2. They have included higher-order terms of the weak nucleon current according to Suhonen, Kadhkikar and Faessler [10] which is based on the simple quark model with a harmonic-oscillator potential for the quarks

Table 2. Comparison of the results of Civitarese and Suhonen (CS) with ours.

Nucleus	CS (QRPA, 1.254)	Our (QRPA, 1.25)
^{76}Ge	3.33	2.68 ± 0.12
^{100}Mo	2.97	1.30 ± 0.10
^{130}Te	3.49	1.56 ± 0.47
^{136}Xe	4.64	0.90 ± 0.20

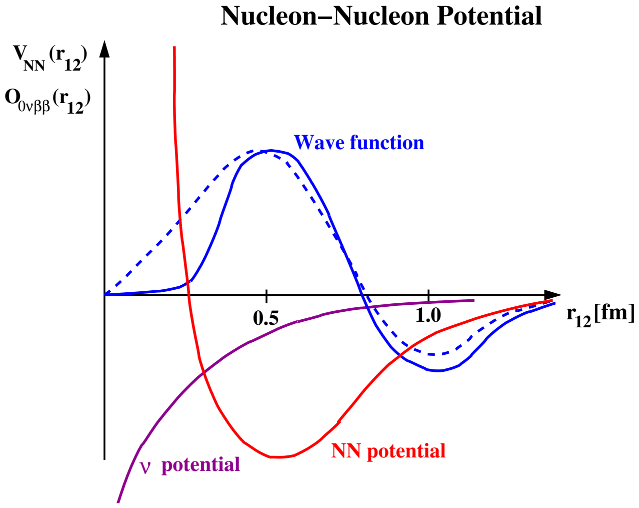

Fig. 6. Qualitative dependence on relative distance r_{12} of the two nucleons involved in the $0\nu\beta\beta$ of the nucleon-nucleon potential, the uncorrelated wave function (dashed line), the correlated wave function and the transition operator (ν potential).

leading to a Gaussian form factor for the weak currents and yielding weak magnetism as the main higher contribution. In our approach we included the higher-order currents according to Towner and Hardy [7] who fitted the higher-order currents to allowed beta decays, to first forbidden beta decays, to beta-gamma angular correlations, to beta-alpha angular correlations and to muon capture in nuclei. Towner and Hardy use dipole form factors and the induced pseudoscalar term and not weak magnetism yields the largest effect.

3. Short-range Brueckner correlations are not included in the calculation of Civitarese and Suhonen [9]. In our calculation, we include short-range Brueckner repulsion between the nucleons for the neutrinoless transition operator in form of a Jastrow factor. The inclusion of the short-range repulsion in the work of Civitarese and Suhonen would reduce the matrix element for the neutrinoless transition probability and could bring it into agreement with our value if also a more accurate expression for the higher-order weak currents and the different adjustment of g_{pp} are included.

Figure 6 shows qualitatively the nucleon-nucleon interaction with the short-range repulsion. In addition the relative wave function of the two nucleons involved in the double-beta decay is given with and without short-range repulsion considered (dashed line). The relative radial dependence of the transition operator for the $0\nu\beta\beta$ decay is also given qualitativeley. The difference in the $0\nu\beta\beta$ decay matrix element with and without short-range correlation is roughly given by the square of the difference between the correlated and uncorrelated wave function weighted by the transition potential for the neutrino (ν potential).

2 Summary

In this contribution we investigated the accuracy of the matrix elements for the neutrinoless double-beta decay. This accuracy determines the accuracy of the Majorana neutrino mass extracted from the neutrinoless double-beta decay. We used the Quasiparticle Random Phase Approach (QRPA) and the renormalized-QRPA (RQRPA) approximation which includes the Pauli principle and reduces the ground-state correlations and stabilises the wave function of the excited states of the intermediate nuclei. To get a feeling for the uncertainty we calculated the neutrinoless double-beta decay for three different nucleon-nucleon forces (Bonn, Nijmegen, Argonne) and three different basis sets (small with about two oscillator shells, intermediate with about three oscillator shells and large with about five oscillator shells). In addition we performed the calculations with QRPA, with R-QRPA (including the Pauli principle) and for two different axial vector coupling constants $g_A = 1.25$ and $g_A = 1.00$. In each of the 36 calculations for every nucleus we adjust a factor g_{pp} multiplying the nucleon-nucleon two-body matrix elements of the order between 0.85 and 1.1 to fit the experimental two-neutrino transition probabilities. We give the theoretical error of the neutrinoless transition probability as the 1σ error of the nine calculations for each method plus the experimental error of the two-neutrino transition probability. In many cases the theoretical error of the neutrinoless nuclear matrix element is less than $\pm 20\%$ but in one case, ^{96}Zr, it is $\pm 100\%$. But the main part of the error as indicated in fig. 5 comes from the experimental error of the two-neutrino transition probability. An improvement of the measurement of the two-neutrino transition probability would even further reduce the uncertainty and by that further reduce the error of the Majorana neutrino mass which can be determined if the neutrinoless double-beta decay is measured.

2.1 Problems to be solved

One open problem is connected with the fact that two-quasi particle excitations of the initial and the final nucleus to the intermediate states yield a slightly different Hilbert space and thus also slightly different intermediate states. This is treated by including an overlap matrix between these states. We are working on a better treatment and methods to see which error is induced by this effect.

A second problem is connected with the proton and neutron number non-conservation in the BCS treatment of pairing. Particle number projection before variation and the use of the Lipkin-Nogami method did show that this is only a problem if one of the nuclei involved has a closed shell [11].

A third problem are nuclear deformations for example for ^{150}Nd [12].

I would like to thank Dr. F. Simkovic, Dr. V. Rodin and Prof. P. Vogel for important contributions to the results presented here.

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